

5/H-24 (v) (Syllabus-2015)

2018

(October)

PHYSICS

(Honours)

(Mathematical Physics, Quantum Mechanics)

[PHY-05 (T)]

Marks : 56

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer Question No. 1 which is compulsory
and any four from the rest

1. (a) Use Gauss' divergence theorem to prove
that $\iiint_S (\nabla\phi \times \nabla\psi) \cdot \hat{n} dS = 0$. 3
- (b) Evaluate (i) $\Gamma(-\frac{1}{2})$ and (ii) $\Gamma(1)$. 3
- (c) Find out the expectation value of the
linear momentum of a particle moving
in a one-dimensional box of width a .
The normalized wave function of the
particle is $\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$. 3

(Turn Over)

(2)

(d) A and B are two Hermitian operators. Prove that AB will also be Hermitian if A and B commute. 3

2. (a) Define a Hermitian matrix. Prove that diagonal elements of a Hermitian matrix are real. 1+2=3 4

(b) Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

(c) Use Stokes' theorem to evaluate $\oint_C \vec{F} \cdot d\vec{R}$ where $\vec{F} = y\hat{i} + xz^3\hat{j} - zy^3\hat{k}$, C is the circle $x^2 + y^2 = 4$ and $z = 1.5$. 4

3. (a) Prove that Kronecker delta (δ_{ij}) is a mixed tensor of rank two. 3

(b) Write the transformation equation of a contravariant tensor. If \dot{x} and \dot{y} are the components of velocity in Cartesian coordinate system, find the components of velocity in polar coordinate system. 5

(c) Prove that $\Gamma(n+1) = n\Gamma(n)$. 3

4. (a) State and prove Cauchy's theorem. 1+3=4

D9/103

(Continued)

(3)

(b) Use Cauchy's integral formula to evaluate

$$\oint_C \frac{\cos \pi z}{z-1} dz$$

where C is the circle $|z|=3$. 3

(c) Use Cauchy's residue theorem to evaluate

$$\oint_C \frac{1-2z}{z(z-1)(z-2)} dz$$

where C is the circle $|z|=1.5\left(\frac{3}{2}\right)$. 4

5. Generating function of Legendre polynomial is given by $(1-2xt+t^2)^{-\frac{1}{2}}$. Use this function to—

(a) find the expression for $P_2(x)$;

(b) prove that $P_n(1) = 1$;

(c) show that $\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}$. 4+3+4=11

6. (a) State and prove Ehrenfest's theorem. 8

(b) Prove that linear momentum operator is a Hermitian operator. 3

D9/103

(Turn Over)

7. (a) A potential step is given by

$$V(x) = 0 \quad x < 0$$

$$= V_0 \quad x \geq 0$$

For a particle of energy $E (< V_0)$ incident from left on the potential step the solutions of Schrödinger equation are given by

$$\psi(x) = A e^{i\alpha x} + B e^{-i\alpha x} \quad \text{for } x < 0$$

$$= C e^{-\beta x} \quad \text{for } x > 0$$

For the system given above, calculate the reflection and transmission coefficient.

- (b) In the region $x > 0$, calculate $\langle x \rangle$, $\langle x^2 \rangle$ and hence prove that $\Delta x \sim \frac{1}{\beta}$.

$$2\frac{1}{2} \times 2 = 5$$

$$2+2+2=6$$

8. (a) Write the eigenvalue equation for L^2 in spherical polar coordinate system. Solve this equation to find an expression for the eigenvalue of L^2 for $L_z = 0$.

$$1+6=7$$

- (b) Prove that while L^2 and L_z are simultaneously measurable with absolute accuracy, L_z and L_x are not measurable with same accuracy level.

$$2+2=4$$