4/EH–29 (iv) (Syllabus–2015)

2019

(April)

MATHEMATICS

(Elective/Honours)

(Statics and Dynamics)

(GHS–41)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* State and prove Lami's theorem. 1+5=6

   (b) If the resultant of two equal forces inclined at an angle 2θ is twice as great as when they are inclined at an angle 2ϕ, prove that $\cos\theta = 2\cos\phi$. 4

   (c) If the magnitudes of two unlike parallel forces P, $Q(P > Q)$ be increased by the same amount, show that the line of action of the resultant will move further off from P. 5

(Turn Over)

2. (a) Three forces P, Q, R acting at the vertices A, B, C respectively of a triangle, each perpendicular to the opposite side, keep it in equilibrium. Prove that $P:Q:R = a:b:c$. 5

(b) $ABCD$ is a rectangle such that $AB = CD = a$ and $BC = DA = b$. Forces P act along AD and CB and forces Q act along AB and CD. Prove that the perpendicular distance between the resultant of the forces P, Q at A and the resultant of the forces P, Q at C is

$$\frac{Pa - Qb}{\sqrt{P^2 + Q^2}}$$ 6

(c) Prove that a force and a couple in the same plane are equivalent to a single force, equal and parallel to the given single force. 4

UNIT—II

3. (a) The moments of a system of coplanar forces acting in the (x, y)-plane about $(0, 0)$, $(a, 0)$, $(0, a)$ are aW, $2aW$, $3aW$ respectively. Find the components parallel to the coordinate axes and the line of action of the single force to which the system is equivalent. 4

(Continued)

(b) Forces P, Q, R, S act along the sides AB, BC, CD, DA of the cyclic quadrilateral $ABCD$, taken in order, where A and B are the extremities of a diameter. If they are in equilibrium, prove that

$$R^2 = P^2 + Q^2 + S^2 + 2PQS/R$$ 5

(c) A body of weight W can just be sustained on a rough inclined plane by a force P, and just dragged up the plane by a force Q, P and Q both acting up the line of the greatest slope. Show that the coefficient of friction is

$$\frac{Q-P}{\sqrt{4W^2 - (P+Q)^2}}$$ 6

4. (a) A uniform ladder rests in limiting equilibrium with its lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical, prove that $\tan\theta = 2\mu$, where μ is the co-efficient of friction. 4

(b) In a uniform circular disc of radius R, a circular hole of radius r is punched out, the distance between the two centres being c, where $r + c < R$. Show that the CG of the remainder is at a distance $\dfrac{cr^2}{R^2 - r^2}$ from the centre of the disc. 5

(Turn Over)

(c) A uniform rod AB is in equilibrium at an angle α with the horizontal, with its upper end A resting against a smooth peg, and its lower end B attached to a light cord which is fastened to a point C on the same level as A. If the cord is inclined to the horizontal at an angle β, then prove that $\tan\beta = 2\tan\alpha + \cot\alpha$. 6

UNIT—III

5. (a) Show that a particle executing SHM requires one-sixth of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude. 4

(b) Show that the time of descent to the centre of force, the force varying inversely as the square of the distance from the centre, through the first half of its initial displacement is to that through the second half as $\pi + 2 : \pi - 2$. 6

(c) If two equal smooth spheres, which are perfectly elastic, impinge at right angles, show that their directions after impact will also be at right angles. 5

6. (a) A ball A of mass m_1 impinges directly on another ball B of mass m_2, which is at rest. After the impact B impinges

(Continued)

directly on a third ball C of mass m_3, which is also at rest. If the velocity imparted to C is the same as A had at first and if all the balls are perfectly elastic, show that 5

$$(m_1 + m_2)(m_2 + m_3) = 4m_1 m_2$$

(b) When the earth attracts a body outside its surface with a force varying inversely as the square of the distance from the centre, a particle is attracted towards the centre of the earth, starting from rest at infinity. Find the velocity of the particle on reaching the earth's surface and also on reaching the centre of the earth. 4+2

(c) A particle is moving in an SHM of amplitude a. Find the new amplitude if the velocity were doubled when the particle is at a distance $\frac{a}{2}$ from the centre, the period remaining unaltered. 4

UNIT—IV

7. (a) A particle moving in a straight line is subject to a resistance which produces a retardation kv^3, where v is the velocity and k is a constant. Show that v and

(Turn Over)

t (the time) are given in terms of s (the distance) by the equations

$$v = \frac{u}{1+ksu}, \; t = \frac{s}{u} + \frac{1}{2}ks^2$$

where u is its initial velocity. Also show that $s = \frac{1}{ku}[\sqrt{1+2ku^2 t} - 1]$. 2+2+2

(b) A particle falls from rest under gravity in a medium whose resistance varies as the square of the velocity. If v be its velocity and v_0 be the velocity which would be acquired if there were no resistance, show that

$$\frac{v^2}{v_0^2} = 1 - \frac{1}{\underline{|2}} \frac{v_0^2}{V^2} + \frac{1}{\underline{|3}} \frac{v_0^4}{V^4} - \dots$$

where V is the terminal velocity. 5

(c) If at any point of the parabolic path the velocity be u and the inclination to the horizontal be θ, show that the particle is moving at right angles to its former direction after a time $\frac{u}{g}\operatorname{cosec}\theta$. 4

8. (a) A particle is projected in a direction making an angle θ with the horizon. If it passes through the point (x_1, y_1), show that $\tan\theta = \frac{y_1}{x_1}\frac{R}{R-x_1}$ where R is the horizontal range. 4

(b) A particle is projected with a velocity $2\sqrt{ag}$, so that it just clears two walls of equal height a which are at a distance $2a$ from each other. Show that the latus rectum of the path is equal to $2a$ and that the time of passing between the walls is $2(\sqrt{a/g})$. 4+1

(c) A particle is projected vertically upwards with velocity u in a medium whose resistance varies as the square of the velocity. Find the greatest height attained by the particle. 6

UNIT—V

9. (a) A particle is moving in a parabola with uniform angular velocity about the focus. Prove that its normal acceleration at any point is proportional to the radius of curvature of its path at that point. 5

(b) A heavy particle of weight W, attached to a fixed point by a light inextensible string, describes a circle in a vertical plane. The tension in the string has values mW and nW respectively when the particle is at the highest and the lowest points of its path. Show that $n = m + 6$. 5

(Turn Over)

(c) Prove that for a system of conservative forces the work done upon the mass m by the forces is equal to the kinetic energy of the mass. 5

10. (a) A shell of mass M is moving with a velocity u in the line AB. An internal explosion, which generates an energy E, breaks it into two masses m_1 and m_2 which moves in the line AB. Show that their velocities are

$$u+\sqrt{\frac{2m_2E}{m_1M}} \quad \text{and} \quad u-\sqrt{\frac{2m_1E}{m_2M}}$$ 5

(b) A particle moves under gravity in a vertical circle, sliding down the convex side of the smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre, show that it will fly off the circle when at a height $\frac{2h}{3}$ above the centre. 6

(c) A heavy particle slides down a smooth cycloid, starting from rest at a cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration is equal to g at every point of the path. 4

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