5/H–29 (vi) (Syllabus–2015)

2019

(October)

MATHEMATICS

(Honours)

(GHS–52)

(Differential Equations and Advanced Dynamics)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write Units I and II together in one answer script and Units III, IV and V together in another answer script

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* Solve :

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 3)y = e^{x^2}$$ 6

(Turn Over)

(b) Solve :

$$\frac{d^2y}{dx^2}-\frac{2}{x}\left(\frac{dy}{dx}\right)+\left(a^2+\frac{2}{x^2}\right)y=0$$

by changing to normal form. 6

(c) Solve :

$$(y+z)dx+(z+x)dy+(x+y)dz=0$$ 3

2. (a) By the method of variation of parameters, solve the equation

$$\frac{d^2y}{dx^2}+4y=4\tan 2x$$ 6

(b) Solve the simultaneous differential equations

$$\frac{dx}{dt}+5x+y=e^t$$

$$\frac{dy}{dt}-x+3y=e^{2t}$$ 6

(c) Solve :

$$\frac{dx}{x^2-yz}=\frac{dy}{y^2-zx}=\frac{dz}{z^2-xy}$$ 3

UNIT—II

(In this unit, $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$)

3. (a) Form a partial differential equation by eliminating the arbitrary functon ϕ from $lx+my+nz=\phi(x^2+y^2+z^2)$. What is the order of this partial differential equation? 6

(b) Solve :

$$(y^2+z^2-x^2)p-2xyq+2zx=0$$ 5

(c) Solve :

$$p^2+q^2=1$$ 4

4. (a) Apply Charpit's method to find complete integral of

$$z^2(p^2z^2+q^2)=1$$ 5

(b) Find the complete integral and singular integral of

$$z=px+qy+p^2+q^2$$ 6

(c) Find the complete integral of

$$\sqrt{p}+\sqrt{q}=2x$$ 4

(Turn Over)

UNIT—III

5. *(a)* A particle moves under a central repulsive force $m\mu/(\text{distance})^3$ and is projected from an apse at a distance a with velocity V. Show that the equation to the path is $r\cos k\theta = a$ and that the angle θ described in time t is $\frac{1}{k}\tan^{-1}\left(\frac{kV}{a}t\right)$ where $k^2 = \frac{\mu + a^2V^2}{a^2V^2}$. 4+4=8

(b) A heavy particle slides down a rough cycloid whose base is horizontal and vertex downwards. Show that if it starts from rest at the cusp and comes to rest at the vertex, then $\mu^2 = e^{-\mu\pi}$. 7

6. *(a)* A particle P, of unit mass, moves under the action of a force of magnitude $\frac{\mu}{SP^2}$, directed towards a fixed point S. If the velocity of P is V when SP is r, then show that the path of P is a conic having S as focus and that the conic is an ellipse, parabola or hyperbola according as V^2 is less than, equal to or greater than $\frac{2\mu}{r}$. 4+3=7

(b) A particle starts from rest from the cusp of a rough cycloid whose axis is

vertical and vertex downwards. Show that its velocity at the vertex to its velocity at the same point when the cycloid is smooth is 8

$$(e^{-\mu\pi} - \mu^2)^{\frac{1}{2}} : (1+\mu^2)^{\frac{1}{2}}$$

UNIT—IV

7. *(a)* Prove that the moment of inertia of a triangular lamina about any line in its plane is given by

$$I = \frac{1}{6}M[h_1^2 + h_2^2 + h_3^2 + h_1h_2 + h_2h_3 + h_3h_1]$$

where M is the mass of the lamina and h_1, h_2, h_3 are respectively the distances of the vertices of the triangle from the line. 8

(b) A uniform solid rectangular block is of mass M and have dimensions $2a$, $2b$, $2c$. Find the equation of the momental ellipsoid for a corner O of the block, referred to edges through O as co-ordinates axes. 7

8. *(a)* The lengths AB and AD of the sides of a rectangle $ABCD$ are $2a$, $2b$; show that the inclination of AB and one of the principal axis at A is 5

$$\frac{1}{2}\tan^{-1}\frac{3ab}{2(a^2 - b^2)}$$

(b) Find the moment of inertia of the uniform triangular lamina ABC, of mass M, about the side BC. 5

(c) Prove that the momental ellipsoid, at the centre of the elliptic plate is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\left(\frac{1}{a^2}+\frac{1}{b^2}\right)z^2=$ constant 5

UNIT—V

9. (a) A perfectly rough circular hoop of radius a rolls on a horizontal floor with velocity U towards an inelastic step of height $h\left(<\frac{a}{2}\right)$, the plane of the hoop being vertical and perpendicular to the edge of the step. Prove that the hoop can mount the step without losing contact at any stage if

$$4a^2hg < U^2(2a-h)^2 < 4a^2(a-h)g$$ 7

(b) A uniform heavy solid hemisphere of radius a is held at rest with its base vertical and its curved surface in contact with a horizontal plane. If the hemisphere is released when the plane is rough enough to prevent slipping, then show that the angle θ that the base makes with the horizontal at time t is such that

$$\left(\frac{d\theta}{dt}\right)^2=\frac{15g\cos\theta}{a(28-15\cos\theta)}$$ 8

10. (a) A uniform rod is placed with one end in contact with a horizontal table and is then at inclination α to the horizontal and is allowed to fall. When it becomes horizontal, show that its angular velocity is $\sqrt{3g\sin\alpha/2a}$, if the table is perfectly rough, show that the rod never leaves the table. 3+3=6

(b) A rigid body of mass M rotates about a horizontal axis through a point O in it. Show that the motion is simple harmonic. Find the period and the length of the equivalent simple pendulum. Show further that the minimum value of the length of the equivalent simple pendulum is $2k$, where k is the radius of gyration of the rigid body. 3+3+3=9

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